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Extracting γ from $\mathrm{B_{s(d)}} \to \mathrm{J}/\psi\mathrm{K_S}$ and $\mathrm{B_{d(s)}} \to \mathrm{D^+_{d(s)}D^-_{d(s)}}$

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Abstract. A completely general parametrization of the time-dependent decay rates of the modes $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$ and $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ is given, which are related to each other through the U-spin flavour symmetry of strong interactions. Owing to the interference of current–current and penguin processes, the $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$ observables probe the angle γ of the unitarity triangle. Using the U-spin symmetry, the overall normalization of the $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$ rate can be fixed with the help of the CP-averaged $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ rate, providing a new strategy to determine γ. This extraction of γ is not affected by any final-state-interaction effects, and its theoretical accuracy is only limited by U-spin-breaking corrections. As a by-product, this strategy allows us to also take into account the penguin effects in the determination of β from $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$, which are presumably very small, and to predict the direct CP asymmetry arising in this mode. An analogous strategy is provided by the time-dependent $\mathrm{B_d} \to \mathrm{D^+D^-}$ rate, if its overall normalization is fixed through the CP-averaged $\mathrm{B_s} \to \mathrm{D_s^+D_s^-}$ rate.

1 Introduction

It is well known that the "gold-plated" mode $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ [1] plays an outstanding role in the determination of $\sin(2\beta)$, where β is one of the three angles α, β and γ of the usual non-squashed unitarity triangle [2] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [3]. First attempts to measure $\sin(2\beta)$ in this way, which is one of the major goals of several B-physics experiments starting very soon, have recently been performed by the OPAL and CDF collaborations [4].

In this paper, we will have a closer look at the general structure of the $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ decay amplitude arising within the Standard Model, and at the one of its U-spin counterpart $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$. The two decays are related to each other by interchanging all down and strange quarks, i.e. through the "U-spin" subgroup of the SU(3) flavour symmetry of strong interactions. Whereas the weak phase factor $e^{i\gamma}$ enters in $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ in a strongly Cabibbo-suppressed way, this is not the case in $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$. Consequently, there may be sizeable CP-violating effects in this $\mathrm{B_s}$ decay, which are due to the interference between current–current and penguin operator contributions. Interestingly, the time evolution of the $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$ decay rate allows us to determine γ. To this end, we have to employ the U-spin symmetry to fix the overall normalization of $\mathrm{B_s} \to \mathrm{J}/\psi\mathrm{K_S}$ through the CP-averaged $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$ rate. This new strategy to extract γ is not affected by QCD or electroweak penguin effects – it rather makes use of these topologies – and does not rely on certain "plausible" dynamical or model-dependent assumptions. Moreover, final-state-interaction effects are taken into account *by definition*, and do not lead to any problems. The theoretical accuracy is only limited by U-spin-breaking corrections. An analogous strategy is provided by the time-dependent $\mathrm{B_d} \to \mathrm{D^+D^-}$ rate, if its overall normalization is fixed through the CP-averaged $\mathrm{B_s} \to \mathrm{D_s^+D_s^-}$ rate, and if the $\mathrm{B_d^0}$–$\overline{\mathrm{B_d^0}}$ mixing phase, i.e. 2β, is determined with the help of $\mathrm{B_d} \to \mathrm{J}/\psi\mathrm{K_S}$.

In particular the determination of γ is an important goal for future B-physics experiments. This angle should be measured in a variety of ways so as to check whether one consistently finds the same result. There are several methods to accomplish this task on the market [5]. Since the $\mathrm{e^+e^-}$ B-factories operating at the $\Upsilon(4S)$ resonance will not be in a position to explore $\mathrm{B_s}$ decays, a strong emphasis has been given to decays of non-strange B mesons in the recent literature. However, also the $\mathrm{B_s}$ system provides interesting strategies to determine γ. In order to make use of these methods, dedicated B-physics experiments at hadron machines, such as LHCb, are the natural place. Within the Standard Model, the weak $\mathrm{B_s^0}$–$\overline{\mathrm{B_s^0}}$ mixing phase is very small, and studies of $\mathrm{B_s}$ decays involve very rapid $\mathrm{B_s^0}$–$\overline{\mathrm{B_s^0}}$ oscillations due to the large mass difference $\Delta M_s \equiv M_H^{(s)} - M_L^{(s)}$ between the mass eigenstates $\mathrm{B_s^H}$ ("heavy") and $\mathrm{B_s^L}$ ("light"). Future B-physics experiments performed at hadron machines should be in a position to resolve these oscillations. Interestingly, in contrast to the $\mathrm{B_d}$ case, there may be a sizeable width difference $\Delta\Gamma_s \equiv \Gamma_H^{(s)} - \Gamma_L^{(s)}$ between the mass eigenstates of the $\mathrm{B_s}$ system [6], which may allow studies of CP violation with

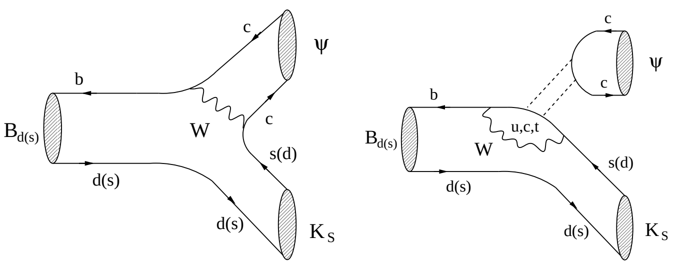

Fig. 1. Feynman diagrams contributing to $B_{d(s)} \to J/\psi K_S$. The dashed lines in the penguin topology represent a colour-singlet exchange

"untagged" B_s data samples, where one does not distinguish between initially, i.e. at time $t = 0$, present B^0_s or $\overline{B^0_s}$ mesons [7]. In such untagged rates, the rapid B^0_s–$\overline{B^0_s}$ oscillations cancel.

Some of the B_s strategies proposed in the literature are theoretically clean, and use pure "tree" decays, for example $B_s \to D^{\pm}_s K^{\mp}$ [8]. Since no flavour-changing neutral-current (FCNC) processes contribute to the corresponding decay amplitudes, it is quite unlikely that they are significantly affected by new physics. Consequently, the preferred mechanism for physics beyond the Standard Model to manifest itself in the corresponding time-dependent decay rates is through contributions to B^0_s–$\overline{B^0_s}$ mixing. In contrast, the decay $B_s \to J/\psi K_S$ discussed in this paper also exhibits CP-violating effects that are due to the interference between "tree" and "penguin", i.e. FCNC, processes. Therefore, new physics may well show up in the corresponding CP asymmetries, thereby affecting the extracted value of γ. A similar comment applies to the $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ strategy.

The outline of this paper is as follows: in Sect. 2, the $B_{d(s)} \to J/\psi K_S$ decay amplitudes are parametrized in a completely general way within the framework of the Standard Model. Moreover, expressions for the observables of the corresponding time-dependent decay rates are given. The strategy to determine γ with the help of these observables is discussed in Sect. 3, whereas we turn to the analogous strategy using $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ decays in Sect. 4. The main results are summarized in Sect. 5.

2 The $B_{d(s)} \to J/\psi K_S$ observables

The decays $B^0_{d(s)} \to J/\psi K_S$ are transitions into a CP eigenstate with eigenvalue -1 and originate from $\bar{b} \to \bar{c}c\bar{s}(\bar{d})$ quark-level decays. We have to deal both with current–current and with penguin contributions, as can be seen in Fig. 1. Let us turn to the mode $B^0_d \to J/\psi K_S$ first. Its transition amplitude can be written as

$$A(B^0_d \to J/\psi K_S) = \lambda_c^{(s)} \left(A_{cc}^{c'} + A_{pen}^{c'} \right) + \lambda_u^{(s)} A_{pen}^{u'} + \lambda_t^{(s)} A_{pen}^{t'}, \quad (1)$$

where $A_{cc}^{c'}$ denotes the current–current contributions, i.e. the "tree" processes in Fig. 1, and the amplitudes $A_{pen}^{q'}$ describe the contributions from penguin topologies with internal q quarks ($q \in \{u, c, t\}$). These penguin amplitudes take into account both QCD and electroweak penguin contributions. The primes in (1) remind us that we are dealing with a $\bar{b} \to \bar{s}$ transition, and

$$\lambda_q^{(s)} \equiv V_{qs} V_{qb}^* \quad (2)$$

are the usual CKM factors. Making use of the unitarity of the CKM matrix and applying the Wolfenstein parametrization [9], generalized to include non-leading terms in λ [10], we obtain

$$A(B^0_d \to J/\psi K_S) = \left(1 - \frac{\lambda^2}{2}\right) \times \mathcal{A}' \left[1 + \left(\frac{\lambda^2}{1-\lambda^2} \right) a' e^{i\theta'} e^{i\gamma} \right], \quad (3)$$

where

$$\mathcal{A}' \equiv \lambda^2 A \left(A_{cc}^{c'} + A_{pen}^{ct'} \right), \quad (4)$$

with $A_{pen}^{ct'} \equiv A_{pen}^{c'} - A_{pen}^{t'}$, and

$$a' e^{i\theta'} \equiv R_b \left(1 - \frac{\lambda^2}{2}\right) \left(\frac{A_{pen}^{ut'}}{A_{cc}^{c'} + A_{pen}^{ct'}} \right). \quad (5)$$

The quantity $A_{pen}^{ut'}$ is defined in analogy to $A_{pen}^{ct'}$, and the relevant CKM factors are given by:

$$\lambda \equiv |V_{us}| = 0.22, \quad A \equiv \frac{1}{\lambda^2} |V_{cb}| = 0.81 \pm 0.06,$$
$$R_b \equiv \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.41 \pm 0.07. \quad (6)$$

The decay $B^0_s \to J/\psi K_S$ is related to $B^0_d \to J/\psi K_S$ by interchanging all down and strange quarks, i.e. through the so-called U-spin subgroup of the SU(3) flavour symmetry of strong interactions. Using again the unitarity of the CKM matrix and a notation similar to that in (3), we obtain

$$A(B^0_s \to J/\psi K_S) = -\lambda \mathcal{A} \left[1 - a e^{i\theta} e^{i\gamma} \right], \quad (7)$$

where

$$\mathcal{A} \equiv \lambda^2 A \left(A_{cc}^{c} + A_{pen}^{ct} \right) \quad (8)$$

and

$$a e^{i\theta} \equiv R_b \left(1 - \frac{\lambda^2}{2}\right) \left(\frac{A_{pen}^{ut}}{A_{cc}^{c} + A_{pen}^{ct}} \right) \quad (9)$$

correspond to (4) and (5), respectively. It should be emphasized that (3) and (7) are completely general parametrizations of the $B^0_{d(s)} \to J/\psi K_S$ decay amplitudes within the Standard Model, relying only on the unitarity of the CKM matrix. In particular, these expressions also take into account final-state-interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges [11,12].

If we compare (3) and (7) with each other, we observe that the quantity $a' e^{i\theta'}$ is doubly Cabibbo-suppressed in the $B^0_d \to J/\psi K_S$ decay amplitude (3), whereas $a e^{i\theta}$ enters in the $B^0_s \to J/\psi K_S$ amplitude (7) in a Cabibbo-allowed way. This feature has important implications for the CP-violating effects arising in the corresponding time-dependent decay rates.

The time evolution for decays of initially, i.e. at time $t = 0$, present neutral B or $\overline{B}$ mesons into a final CP eigenstate $|f\rangle$, satisfying

$$(\mathcal{CP})|f\rangle = \eta|f\rangle, \tag{10}$$

is given as follows [5]:

$$\begin{aligned}|A(t)|^2 = \frac{|\mathcal{N}|^2}{2}\big[&R_L e^{-\Gamma_L t} + R_H e^{-\Gamma_H t}\\ &+2e^{-\Gamma t}\{A_D \cos(\Delta M t)\\ &+A_M \sin(\Delta M t)\}\big],\end{aligned} \tag{11}$$

$$\begin{aligned}|\overline{A}(t)|^2 = \frac{|\mathcal{N}|^2}{2}\big[&R_L e^{-\Gamma_L t} + R_H e^{-\Gamma_H t}\\ &-2e^{-\Gamma t}\{A_D \cos(\Delta M t)\\ &+A_M \sin(\Delta M t)\}\big],\end{aligned} \tag{12}$$

where the $\Gamma_{L,H}$ denote the decay widths of the B mass eigenstates, $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$, and $\Delta M \equiv M_H - M_L > 0$ is their mass difference. For the B decays considered in this paper, the "unevolved" decay amplitudes take the form

$$A = \mathcal{N}\left[1 - b e^{i\rho} e^{+i\gamma}\right] \equiv \mathcal{N} z, \tag{13}$$

$$\overline{A} = \eta\mathcal{N}\left[1 - b e^{i\rho} e^{-i\gamma}\right] \equiv \eta\mathcal{N}\overline{z}, \tag{14}$$

and we have

$$\begin{aligned}R_L &\equiv \frac{1}{2}\left[|z|^2 + |\overline{z}|^2 + 2\eta\Re\left(e^{-i\phi} z^* \overline{z}\right)\right]\\ &= (1 + \eta\cos\phi) - 2b\cos\rho\left[\cos\gamma + \eta\cos(\phi+\gamma)\right]\\ &\quad + b^2\left[1 + \eta\cos(\phi + 2\gamma)\right],\end{aligned} \tag{15}$$

$$\begin{aligned}R_H &\equiv \frac{1}{2}\left[|z|^2 + |\overline{z}|^2 - 2\eta\Re\left(e^{-i\phi} z^* \overline{z}\right)\right]\\ &= (1 - \eta\cos\phi) - 2b\cos\rho\left[\cos\gamma - \eta\cos(\phi+\gamma)\right]\\ &\quad + b^2\left[1 - \eta\cos(\phi + 2\gamma)\right],\end{aligned} \tag{16}$$

$$A_D \equiv \frac{1}{2}\left(|z|^2 - |\overline{z}|^2\right) = 2b\sin\rho\sin\gamma, \tag{17}$$

$$\begin{aligned}A_M &\equiv -\eta\Im\left(e^{-i\phi} z^* \overline{z}\right)\\ &= \eta\left[\sin\phi - 2b\cos\rho\sin(\phi+\gamma) + b^2\sin(\phi+2\gamma)\right]\end{aligned} \tag{18}$$

Here the phase ϕ denotes the B–$\overline{B}$ mixing phase:

$$\phi = \begin{cases} 2\beta & B_d \text{ system},\\ -2\delta\gamma & B_s \text{ system},\end{cases} \tag{19}$$

where $2\delta\gamma \approx 0.03$ is tiny in the Standard Model because of a Cabibbo suppression of $\mathcal{O}(\lambda^2)$. Note that the observables R_L, R_H, A_D and A_M satisfy the relation

$$A_D^2 + A_M^2 = R_L R_H. \tag{20}$$

For the following considerations, it is useful to introduce the time-dependent CP asymmetry

$$\begin{aligned}a_{CP}(t) &\equiv \frac{|A(t)|^2 - |\overline{A}(t)|^2}{|A(t)|^2 + |\overline{A}(t)|^2}\\ &= 2e^{-\Gamma t}\left[\frac{\mathcal{A}_{CP}^{dir}\cos(\Delta M t) + \mathcal{A}_{CP}^{mix}\sin(\Delta M t)}{e^{-\Gamma_H t} + e^{-\Gamma_L t} + \mathcal{A}_{\Delta\Gamma}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)}\right]\end{aligned} \tag{21}$$

with

$$\mathcal{A}_{CP}^{dir} \equiv \frac{2A_D}{R_H + R_L} = \frac{2b\sin\rho\sin\gamma}{1 - 2b\cos\rho\cos\gamma + b^2}, \tag{22}$$

$$\begin{aligned}\mathcal{A}_{CP}^{mix} &\equiv \frac{2A_M}{R_H + R_L}\\ &= +\eta\left[\frac{\sin\phi - 2b\cos\rho\sin(\phi+\gamma) + b^2\sin(\phi+2\gamma)}{1 - 2b\cos\rho\cos\gamma + b^2}\right],\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{A}_{\Delta\Gamma} &\equiv \frac{R_H - R_L}{R_H + R_L}\\ &= -\eta\left[\frac{\cos\phi - 2b\cos\rho\cos(\phi+\gamma) + b^2\cos(\phi+2\gamma)}{1 - 2b\cos\rho\cos\gamma + b^2}\right],\end{aligned} \tag{24}$$

and the observable

$$R \equiv \frac{1}{2}\left(R_H + R_L\right) = 1 - 2b\cos\rho\cos\gamma + b^2. \tag{25}$$

In the CP asymmetry (21), we have separated the "direct" from the "mixing-induced" CP-violating contributions. It is interesting to note that not only $\mathcal{A}_{CP}^{dir}$, but also R does not depend on the B–$\overline{B}$ mixing phase ϕ. The observables $\mathcal{A}_{CP}^{dir}$, $\mathcal{A}_{CP}^{mix}$ and $\mathcal{A}_{\Delta\Gamma}$ are not independent quantities, and satisfy the relation

$$(\mathcal{A}_{CP}^{dir})^2 + (\mathcal{A}_{CP}^{mix})^2 + (\mathcal{A}_{\Delta\Gamma})^2 = 1. \tag{26}$$

The formulae given above describe the time evolution of all kinds of neutral B decays into a final CP eigenstate, where the "unevolved" decay amplitudes take the form specified in (13) and (14). Let us turn, in the following section, to the $B_{s(d)} \to J/\psi K_S$ observables, which may provide an interesting strategy to determine γ.

3 Extracting γ from $B_{s(d)} \to J/\psi K_S$ decays

The observables introduced in (22)–(24) can be obtained directly from the time evolution of the decay rates corresponding to (11) and (12) and do not depend on the overall normalization $|\mathcal{N}|^2$. However, owing to (26), we have only two independent observables, depending on the three "unknowns" b, ρ and γ, and on the B–$\overline{B}$ mixing phase ϕ. Consequently, in order to determine these "unknowns", we need an additional observable, which is provided by R. Unfortunately, the time-dependent decay rates fix only the quantity

$$\langle\Gamma\rangle \equiv \text{PhSp} \times |\mathcal{N}|^2 \times R = \text{PhSp} \times |\mathcal{N}|^2 \times \frac{1}{2}(R_H + R_L) \tag{27}$$

through

$$\Gamma(B(t)\to f)+\Gamma(\overline{B}(t)\to f) = \mathrm{PhSp}\times|\mathcal{N}|^2\times\left[R_{\rm H}e^{-\Gamma_{\rm H}t}+R_{\rm L}e^{-\Gamma_{\rm L}t}\right], \quad (28)$$

where PhSp denotes an appropriate, straightforwardly calculable phase-space factor. Consequently, the overall normalization $|\mathcal{N}|^2$ is required in order to determine R. In the case of the decay $B_s \to J/\psi K_S$, this normalization can be fixed through the CP-averaged $B_d \to J/\psi K_S$ rate with the help of the U-spin symmetry.

In the case of $B_d \to J/\psi K_S$, we have

$$\mathcal{N}=\left(1-\frac{\lambda^2}{2}\right)\mathcal{A}', \quad b=\epsilon a', \quad \rho=\theta'+180^\circ, \quad \text{with} \quad \epsilon\equiv\frac{\lambda^2}{1-\lambda^2}, \quad (29)$$

whereas we have in the $B_s \to J/\psi K_S$ case

$$\mathcal{N}=-\lambda\mathcal{A}, \quad b=a, \quad \rho=\theta. \quad (30)$$

Consequently, we obtain

$$H\equiv\frac{1}{\epsilon}\left(\frac{|\mathcal{A}'|}{|\mathcal{A}|}\right)^2\left[\frac{M_{B_d}\Phi(M_{J/\psi}/M_{B_d},M_K/M_{B_d})}{M_{B_s}\Phi(M_{J/\psi}/M_{B_s},M_K/M_{B_s})}\right]^3\frac{\langle\Gamma\rangle}{\langle\Gamma'\rangle} = \frac{1-2a\cos\theta\cos\gamma+a^2}{1+2\epsilon a'\cos\theta'\cos\gamma+\epsilon^2a'^2}, \quad (31)$$

where

$$\Phi(x,y)=\sqrt{[1-(x+y)^2]\,[1-(x-y)^2]} \quad (32)$$

is the usual two-body phase-space function, and $\langle\Gamma\rangle \equiv \langle\Gamma(B_s \to J/\psi K_S)\rangle$ and $\langle\Gamma'\rangle \equiv \langle\Gamma(B_d \to J/\psi K_S)\rangle$ can be determined from the "untagged" $B_{s(d)} \to J/\psi K_S$ rates with the help of (27) and (28). Since the U-spin flavour symmetry of strong interactions implies

$$|\mathcal{A}'|=|\mathcal{A}| \quad (33)$$

and

$$a'=a, \quad \theta'=\theta, \quad (34)$$

we can determine a, θ and γ as a function of the B^0_s–$\overline{B^0_s}$ mixing phase by combining H with $\mathcal{A}^{\rm dir}_{\rm CP} \equiv \mathcal{A}^{\rm dir}_{\rm CP}(B_s \to J/\psi K_S)$ and $\mathcal{A}^{\rm mix}_{\rm CP} \equiv \mathcal{A}^{\rm mix}_{\rm CP}(B_s \to J/\psi K_S)$ or $\mathcal{A}_{\Delta\Gamma} \equiv \mathcal{A}_{\Delta\Gamma}(B_s \to J/\psi K_S)$. In contrast to certain isospin relations, electroweak penguins do not lead to any problems in these U-spin relations. As we have already noted, the B^0_s–$\overline{B^0_s}$ mixing phase $\phi=-2\delta\gamma$ is expected to be negligibly small in the Standard Model. It can be probed with the help of the decay $B_s \to J/\psi\phi$ (see, for example, [13]). Large CP-violating effects in this decay would signal that $2\delta\gamma$ is not tiny, and would indicate new-physics contributions to B^0_s–$\overline{B^0_s}$ mixing. Strictly speaking, in the case of $B_s \to J/\psi K_S$, we have $\phi=-2\delta\gamma-\phi_K$, where ϕ_K is related to the K^0–$\overline{K^0}$ mixing phase and is negligibly small in the Standard Model. On the other hand, we have $\phi=2\beta+\phi_K$ in the case of $B_d \to J/\psi K_S$. Since the value of the CP-violating parameter ε_K of the neutral kaon system is small, ϕ_K can only be affected by very contrived models of new physics [14].

An important by-product of the strategy described above is that the quantities a' and θ' allow us to take into account the penguin contributions in the determination of β from $B_d \to J/\psi K_S$, which are presumably very small because of the Cabibbo suppression of $\lambda^2/(1-\lambda^2)$ in (3). Moreover, using (34), we obtain an interesting relation between the direct CP asymmetries arising in the modes $B_d \to J/\psi K_S$ and $B_s \to J/\psi K_S$ and their CP-averaged rates:

$$\frac{\mathcal{A}^{\rm dir}_{\rm CP}(B_d\to J/\psi K_S)}{\mathcal{A}^{\rm dir}_{\rm CP}(B_s\to J/\psi K_S)}=-\epsilon H = -\left(\frac{|\mathcal{A}'|}{|\mathcal{A}|}\right)^2\left[\frac{M_{B_d}\Phi(M_{J/\psi}/M_{B_d},M_K/M_{B_d})}{M_{B_s}\Phi(M_{J/\psi}/M_{B_s},M_K/M_{B_s})}\right]^3\frac{\langle\Gamma\rangle}{\langle\Gamma'\rangle}. \quad (35)$$

An analogous relation holds also between the $B^\pm \to \pi^\pm K$ and $B^\pm \to K^\pm K$ CP-violating asymmetries [11,12]. At "second-generation" B-physics experiments at hadron machines, for instance at LHCb, the sensitivity may be good enough to resolve a direct CP asymmetry in $B_d \to J/\psi K_S$. In view of the impressive accuracy that can be achieved in the era of such experiments, it is also an important issue to think about the theoretical accuracy of the determination of β from $B_d \to J/\psi K_S$. The approach discussed above allows us to control these – presumably very small – hadronic uncertainties with the help of $B_s \to J/\psi K_S$.

Interestingly, the strategy to extract γ from $B_{s(d)} \to J/\psi K_S$ decays does not require a non-trivial CP-conserving strong phase θ. However, its experimental feasibility depends strongly on the value of the quantity a introduced in (9). It is very difficult to estimate a theoretically. In contrast to the "usual" QCD penguin topologies, the QCD penguins contributing to $B_{s(d)} \to J/\psi K_S$ require a colour-singlet exchange, as indicated in Fig. 1 through the dashed lines, and are "Zweig-suppressed". Such a comment does not apply to the electroweak penguins, which contribute in "colour-allowed" form. The current–current amplitude $A^{\rm c}_{\rm cc}$ is due to "colour-suppressed" topologies, and the ratio $A^{\rm ut}_{\rm pen}/(A^{\rm c}_{\rm cc}+A^{\rm ct}_{\rm pen})$, which governs a, may be sizeable. It is interesting to note that the measured branching ratio $\mathrm{BR}(B^0_d \to J/\psi K^0) = 2\mathrm{BR}(B^0_d \to J/\psi K_S) = (8.9\pm1.2)\times 10^{-4}$ [15] probes only the combination $\mathcal{A}' \propto \left(A^{\rm c'}_{\rm cc}+A^{\rm ct'}_{\rm pen}\right)$ of current–current and penguin amplitudes, and obviously does not allow us to separate these contributions. It would be very important to have a better theoretical understanding of the quantity $ae^{i\theta}$. However, such analyses are far beyond the scope of this paper, and are left for further studies. If we use

$$\frac{\mathrm{BR}(B_s\to J/\psi K_S)}{\mathrm{BR}(B_d\to J/\psi K_S)}=\epsilon H\left(\frac{|\mathcal{A}|}{|\mathcal{A}'|}\right)^2 \times\left[\frac{M_{B_s}\Phi(M_{J/\psi}/M_{B_s},M_K/M_{B_s})}{M_{B_d}\Phi(M_{J/\psi}/M_{B_d},M_K/M_{B_d})}\right]^3\frac{\tau_{B_s}}{\tau_{B_d}} \quad (36)$$

and (33), we expect a $B_s \to J/\psi K_S$ branching ratio at the level of 2×10^{-5}.

The general expressions for the observables (22)–(24) and (31) are quite complicated. However, they simplify considerably if we keep only the terms linear in a. Within this approximation, we obtain the simple result

$$\tan\gamma \approx \frac{\sin\phi - \eta\mathcal{A}_{\rm CP}^{\rm mix}}{(1-H)\cos\phi} = -\left.\left(\frac{\eta\mathcal{A}_{\rm CP}^{\rm mix}}{1-H}\right)\right|_{\phi=0}, \qquad (37)$$

allowing us to determine γ from the CP-averaged $B_{s(d)} \to J/\psi K_S$ rates and the mixing-induced CP asymmetry arising in $B_s \to J/\psi K_S$.

In the general case, where no approximations are made, there is also a "transparent" strategy to determine γ. The point is that the CP-violating asymmetries $\mathcal{A}_{\rm CP}^{\rm dir}$ and $\mathcal{A}_{\rm CP}^{\rm mix}$ allow us to fix contours in the γ–a plane, which are described by

$$a = \sqrt{\frac{1}{k}\left[l \pm \sqrt{l^2 - hk}\right]}, \qquad (38)$$

where

$$h = u^2 + D(1 - u\cos\gamma)^2, \qquad (39)$$
$$k = v^2 + D(1 - v\cos\gamma)^2, \qquad (40)$$
$$l = 2 - uv - D(1 - u\cos\gamma)(1 - v\cos\gamma), \qquad (41)$$

with

$$u = \frac{(\eta\mathcal{A}_{\rm CP}^{\rm mix}) - \sin\phi}{(\eta\mathcal{A}_{\rm CP}^{\rm mix})\cos\gamma - \sin(\phi+\gamma)}, \qquad (42)$$
$$v = \frac{(\eta\mathcal{A}_{\rm CP}^{\rm mix}) - \sin(\phi+2\gamma)}{(\eta\mathcal{A}_{\rm CP}^{\rm mix})\cos\gamma - \sin(\phi+\gamma)} \qquad (43)$$

and

$$D = \left(\frac{\mathcal{A}_{\rm CP}^{\rm dir}}{\sin\gamma}\right)^2. \qquad (44)$$

It should be emphasized that these contours are *theoretically clean*. It is also possible to combine the direct and mixing-induced CP asymmetries arising in $B_d \to \pi^+\pi^-$ in an analogous way [16], allowing us to fix certain contours as well [17].

So far, we have not yet used the observable H. Combining it with $\mathcal{A}_{\rm CP}^{\rm mix}$, we can fix another contour in the γ–a plane:

$$a = \sqrt{\frac{H - 1 + u(1+\epsilon H)\cos\gamma}{1 - v(1+\epsilon H)\cos\gamma - \epsilon^2 H}}. \qquad (45)$$

If we use $\mathcal{A}_{\Delta\Gamma}$ instead of $\mathcal{A}_{\rm CP}^{\rm mix}$, we obtain the same expression for a as given in (45), where u and v specified in (42) and (43) are replaced by

$$u \to \frac{(\eta\mathcal{A}_{\Delta\Gamma}) + \cos\phi}{(\eta\mathcal{A}_{\Delta\Gamma})\cos\gamma + \cos(\phi+\gamma)}, \qquad (46)$$
$$v \to \frac{(\eta\mathcal{A}_{\Delta\Gamma}) + \cos(\phi+2\gamma)}{(\eta\mathcal{A}_{\Delta\Gamma})\cos\gamma + \cos(\phi+\gamma)}. \qquad (47)$$

The intersection of the contours described by (38) and (45) fixes both a and γ. Let us illustrate this approach in a quantitative way by considering a simple example. Assuming a negligible B_s^0–$\overline{B_s^0}$ mixing phase, i.e. $\phi = 0$, and $\gamma = 76°$, which lies within the range allowed at present for this angle, implied by the usual indirect fits of the unitarity triangle, as well as $a = a' = 0.2$ and $\theta = \theta' = 30°$, we obtain the $B_s \to J/\psi K_S$ observables $\mathcal{A}_{\rm CP}^{\rm dir} = 0.20$, $\mathcal{A}_{\rm CP}^{\rm mix} = 0.33$, $\mathcal{A}_{\Delta\Gamma} = 0.92$ and $H = 0.95$. The corresponding contours in the γ–a plane are shown in Fig. 2, where the solid lines are obtained with the help of (38), and the dot-dashed lines correspond to (45). Interestingly, in the case of the contours shown in Fig. 2, we would not have to deal with "physical" discrete ambiguities for γ, since values of a larger than 1 would simply appear unrealistic. If it should become possible to measure $\mathcal{A}_{\Delta\Gamma}$ with the help of the widths difference $\Delta\Gamma_s$, the dotted line could be fixed. In this example, the approximate expression (37) yields $\gamma \approx 82°$, which deviates from the "true" value of $\gamma = 76°$ by only 8%. It is also interesting to note that we have $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to J/\psi K_S) = -0.98\%$ in our example.

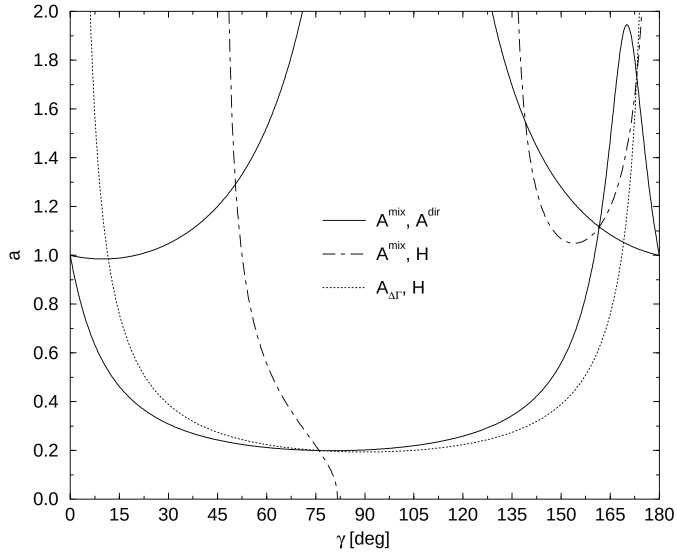

Fig. 2. The contours in the γ–a plane fixed through the $B_{s(d)} \to J/\psi K_S$ observables for a specific example discussed in the text

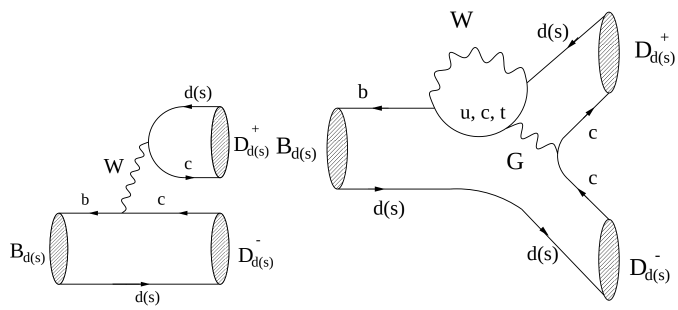

Fig. 3. Feynman diagrams contributing to $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$

Before turning to the $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ decays in the next section, let us say a few words on the SU(3)-breaking corrections. Whereas the contours in the γ–a plane related to (38), i.e. the solid curves in Fig. 2, are *theoret-*

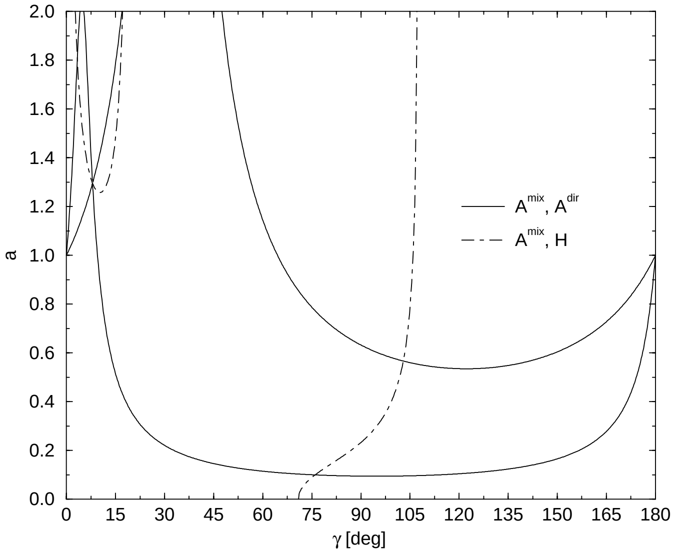

Fig. 4. The contours in the γ–$\tilde{a}$ plane fixed through the $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ observables for a specific example discussed in the text ($2\beta = 53°$)

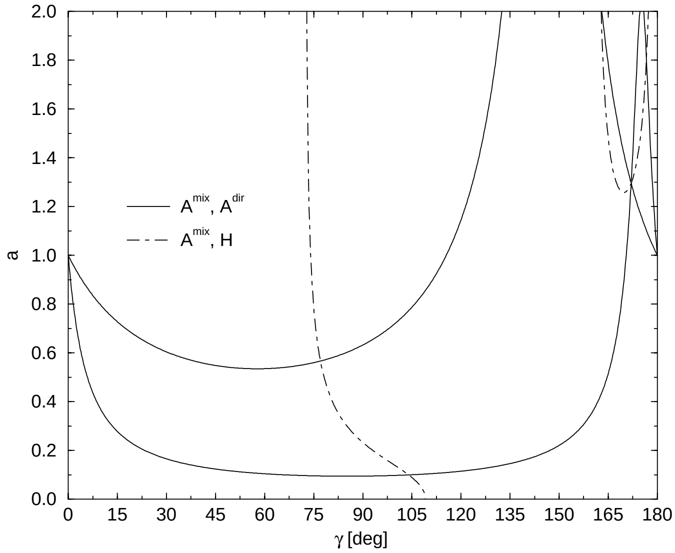

Fig. 5. The contours in the γ–$\tilde{a}$ plane fixed through the $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ observables for a specific example discussed in the text ($2\beta = 180° - 53°$)

ically clean, those described by (45), i.e. the dot-dashed and dotted lines in Fig. 2, are affected by U-spin-breaking corrections. Because of the small parameter $\epsilon = 0.05$ in (31), these contours are essentially unaffected by possible corrections to (34), and rely predominantly on the U-spin relation $|\mathcal{A}'| = |\mathcal{A}|$. In the "factorization" approximation, we have

$$\left.\frac{|\mathcal{A}'|}{|\mathcal{A}|}\right|_{\rm fact} = \frac{F_{B^0_d K^0}(M^2_{J/\psi}; 1^-)}{F_{B^0_s \overline{K^0}}(M^2_{J/\psi}; 1^-)}, \tag{48}$$

where the form factors $F_{B^0_d K^0}(M^2_{J/\psi}; 1^-)$ and $F_{B^0_s \overline{K^0}}(M^2_{J/\psi}; 1^-)$ parametrize the quark-current matrix elements $\langle K^0 | (\bar{b}s)_{V-A} | B^0_d \rangle$ and $\langle \overline{K^0} | (\bar{b}d)_{V-A} | B^0_s \rangle$, respectively [18]. We are not aware of quantitative studies of (48), which could be performed, for instance, with the help of sum-rule or lattice techniques. In the light-cone sum-rule approach, sizeable SU(3)-breaking effects were found in the case of the $B_{d,s} \to K^*$ form factors [19]. It should be emphasized that also non-factorizable corrections, which are not included in (48), may play an important role. We are optimistic that we will have a better picture of SU(3) breaking by the time the $B_s \to J/\psi K_S$ measurements can be performed in practice.

4 Extracting γ from $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ decays

The decays $B^0_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ are transitions into a CP eigenstate with eigenvalue +1 and originate from $\bar{b} \to \bar{c}c\bar{d}(\bar{s})$ quark-level decays. We have to deal both with current–current and with penguin contributions, as can be seen in Fig. 3. In analogy to (3) and (7), the corresponding transition amplitudes can be written as

$$A(B^0_s \to D^+_s D^-_s) = \left(1 - \frac{\lambda^2}{2}\right) \times \tilde{\mathcal{A}}' \left[1 + \left(\frac{\lambda^2}{1-\lambda^2}\right) \tilde{a}' e^{i\tilde{\theta}'} e^{i\gamma}\right] \tag{49}$$

$$A(B^0_d \to D^+_d D^-_d) = -\lambda \tilde{\mathcal{A}} \left[1 - \tilde{a} e^{i\tilde{\theta}} e^{i\gamma}\right], \tag{50}$$

where the quantities $\tilde{\mathcal{A}}$, $\tilde{\mathcal{A}}'$ and $\tilde{a}e^{i\tilde{\theta}}$, $\tilde{a}'e^{i\tilde{\theta}'}$ take the same form as in the $B_{s(d)} \to J/\psi K_S$ case. In contrast to $B_{s(d)} \to J/\psi K_S$, there are "colour-allowed" current–current contributions to $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$, as well as contributions from "exchange" topologies, and the QCD penguins do not require a colour-singlet exchange, i.e. are not "Zweig-suppressed".

Usually, $B_d \to D^+_d D^-_d$ decays appear in the literature as a tool to probe β [5]. In fact, if penguins played a negligible role in these modes, β could be determined from the corresponding mixing-induced CP-violating effects. However, the penguin topologies, which contain also important contributions from final-state-interaction effects, may well be sizeable, although it is very difficult to calculate them in a reliable way. The strategy proposed here makes use of these penguin topologies, allowing us to determine γ, if the overall $B_d \to D^+_d D^-_d$ normalization is fixed through the CP-averaged, i.e. the "untagged" $B_s \to D^+_s D^-_s$ rate, and if the B^0_d–$\overline{B^0_d}$ mixing phase 2β is determined with the help of $B_d \to J/\psi K_S$. It should be emphasized that no $\Delta M_s t$ oscillations have to be resolved to measure the untagged $B_s \to D^+_s D^-_s$ rate. Since the phase structures of the $B^0_d \to D^+_d D^-_d$ and $B^0_s \to D^+_s D^-_s$ decay amplitudes are completely analogous to those of $B^0_s \to J/\psi K_S$ and $B^0_d \to J/\psi K_S$, respectively, the formalism developed in the previous section can be applied by performing straightforward replacements of variables. Taking into account phase-space effects, we have

$$\tilde{H} = \frac{1}{\epsilon}\left(\frac{|\tilde{\mathcal{A}}'|}{|\tilde{\mathcal{A}}|}\right)^2 \left[\frac{M_{B_d}}{M_{B_s}} \frac{\Phi(M_{D_s}/M_{B_s}, M_{D_s}/M_{B_s})}{\Phi(M_{D_d}/M_{B_d}, M_{D_d}/M_{B_d})}\right] \frac{\langle\tilde{\Gamma}\rangle}{\langle\tilde{\Gamma}'\rangle}, \tag{51}$$

where the CP-averaged rates $\langle\tilde{\Gamma}\rangle \equiv \langle\Gamma(B_d \to D^+_d D^-_d)\rangle$ and $\langle\tilde{\Gamma}'\rangle \equiv \langle\Gamma(B_s \to D^+_s D^-_s)\rangle$ can be determined with the help of (27) and (28), and the function $\Phi(x, y)$ is as given in (32).

Let us illustrate the strategy to determine γ, again by considering a simple example. Assuming $\tilde{a} = \tilde{a}' = 0.1$, $\tilde{\theta} = \tilde{\theta}' = 210°$, $\gamma = 76°$ and a B^0_d–$\overline{B^0_d}$ mixing phase of $\phi = 2\beta = 53°$, we obtain the $B_d \to D^+_d D^-_d$ observables $\tilde{\mathcal{A}}^{\rm dir}_{\rm CP} = -0.092$, $\tilde{\mathcal{A}}^{\rm mix}_{\rm CP} = 0.88$ and $\tilde{H} = 1.05$. In this case, studies of CP violation in $B_d \to J/\psi K_S$ would yield $\sin(2\beta) = 0.8$, which is the central value of the most recent CDF analysis [4], implying $2\beta = 53°$ or $2\beta = 180° - 53° = 127°$. The former solution for 2β would lead to the contours in the γ–$\tilde{a}$ plane shown in Fig. 4. The contours corresponding to $2\beta = 127°$ are shown in Fig. 5. Since values of $\tilde{a} = \mathcal{O}(1)$ appear unrealistic, we would obtain the two "physical" solutions of 76° and 104° for γ, which are due to the twofold ambiguity of 2β. There are several strategies to resolve this discrete ambiguity in the extraction of 2β [20], which should be feasible in the era of "second-generation" B-physics experiments.

As in the $B_{s(d)} \to J/\psi K_S$ case, only the contours involving the observable $\tilde{H}$ are affected by SU(3)-breaking corrections. Because of the small parameter ϵ, they are essentially due to the U-spin-breaking corrections to $|\tilde{\mathcal{A}}'| = |\tilde{\mathcal{A}}|$. Within the "factorization" approximation, we have

$$\left.\frac{|\tilde{\mathcal{A}}'|}{|\tilde{\mathcal{A}}|}\right|_{\rm fact} \approx \frac{(M_{B_s} - M_{D_s})\sqrt{M_{B_s} M_{D_s}}(w_s + 1)}{(M_{B_d} - M_{D_d})\sqrt{M_{B_d} M_{D_d}}(w_d + 1)} \frac{f_{D_s} \xi_s(w_s)}{f_{D_d} \xi_d(w_d)}, \tag{52}$$

where the restrictions form the heavy-quark effective theory for the $B_q \to D_q$ form factors have been taken into account by introducing appropriate Isgur–Wise functions $\xi_q(w_q)$ with $w_q = M_{B_q}/(2M_{D_q})$ [21]. Studies of the light-quark dependence of the Isgur–Wise function were performed within heavy-meson chiral perturbation theory, indicating an enhancement of ξ_s/ξ_d at the level of 5% [22]. Applying the same formalism to f_{D_s}/f_D gives values at the 1.2 level [23], which is of the same order of magnitude as the results of recent lattice calculations [24]. Further studies are needed to get a better picture of the SU(3)-breaking corrections to the ratio $|\tilde{\mathcal{A}}'|/|\tilde{\mathcal{A}}|$. Since "factorization" may work reasonably well for $B_q \to D^+_q D^-_q$, the leading corrections are expected to be due to (52).

The experimental feasibility of the strategy to extract γ from $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ decays depends strongly on the size of the penguin parameter $\tilde{a}$, which is difficult to predict theoretically. The branching ratio for $B^0_d \to D^+_d D^-_d$ is expected at the 4×10^{-4} level [21]; the one for $B^0_s \to D^+_s D^-_s$ is enhanced by $1/\epsilon = 20$, and is correspondingly expected at the 8×10^{-3} level. Already at the asymmetric e^+e^- B-factories starting very soon, it should be possible to perform time-dependent measurements of the decay $B_d \to D^+_d D^-_d$, whereas $B_s \to D^+_s D^-_s$ – and its "untagged" rate – may be accessible at CDF or HERA-B. However, unless the penguin effects in $B_d \to D^+_d D^-_d$ are very large, the approach to determine γ discussed in this section appears to be particularly interesting for "second-generation" experiments, such as LHCb. The e^+e^- B-factory experiments should nevertheless have a very careful look at the decay $B_d \to D^+_d D^-_d$, and those at hadron machines should study its U-spin counterpart $B_s \to D^+_s D^-_s$.

5 Summary

The observables of the time-dependent $B_s \to J/\psi K_S$ rate, in combination with the CP-averaged $B_d \to J/\psi K_S$ rate, provide an interesting strategy to determine the angle γ of the unitarity triangle. This approach is not affected by any final-state-interaction effects, and its theoretical accuracy is only limited by the U-spin flavour symmetry of strong interactions. As a by-product, it allows us to take into account the penguin effects in the determination of β from $B_d \to J/\psi K_S$, which are presumably very small. An analogous strategy is provided by the time evolution of $B_d \to D^+_d D^-_d$ decays and the untagged $B_s \to D^+_s D^-_s$ rate.

These new strategies may be promising for "second-generation" B-physics experiments, for example LHCb. Their experimental feasibility strongly depends on the size of the penguin effects in $B_{s(d)} \to J/\psi K_S$ and $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$, which are very difficult to calculate and require further theoretical studies. Recent experimental results of the CLEO collaboration on certain non-leptonic B decays, which are dominated by penguin contributions, have shown that these topologies may well lead to surprises.

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